

# Scheduling a Single Mobile Robot for Feeding Tasks in a Manufacturing Cell

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**Abstract.** This paper deals with the problem of finding optimal feeding sequence in a manufacturing cell with feeders fed by a mobile robot with manipulation arm. The performance criterion is to minimize total traveling time of the robot in a given planning horizon. Besides, the robot has to be scheduled in order to keep production lines within the cell working without any shortage of parts fed from feeders. A mixed-integer linear programming (MILP) model is developed to find the optimal solution for the problem. In the MILP formulation, a method of maximum and minimum levels for feeders, inspired by the  $(s, Q)$  inventory system, is applied to define time window for each feeding task. A case study is implemented at the impeller production line to demonstrate the result of the proposed MILP model.

Keywords: Scheduling, Mobile Robot, Feeding Sequence, MILP

## 1 Introduction

Today's production systems range from fully automated to strictly manual. While the former is very efficient in high volumes but less flexible, the latter is very flexible but less cost-efficient in high wage countries. Therefore, manufactures visualize the need for transformable production systems that combines the best of both worlds by using new assistive automation and mobile robots. A given problem is particularly considered for mobile robots with manipulation arms which will automate extended logistic tasks by not only transporting but also collecting containers of parts and emptying them into the place needed. Feeding operation studied in this paper is a kind of extended logistic tasks. However, to utilize mobile robots in an efficient manner requires the ability to properly schedule these feeding tasks. Hence, it is important to plan in which sequence mobile robots process feeding operations so that they could effectively work while satisfying a number of technological constraints.

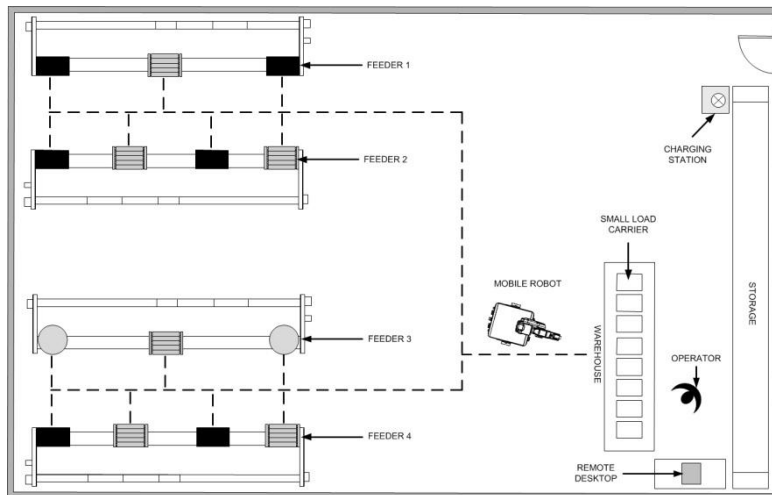
Robot scheduling problem which is NP-hard has attracted interest of researchers in recent decades. Dror and Stulman [1] dealt with the problem of optimizing one-dimensional robot's service movements. Crama and van de Klundert [2] considered the flow shop problem with one transporting robot and one type of product to find

shortest cyclic schedule for the robot. Afterwards, they demonstrated that the sequence of activities whose execution produces one part yields optimal production rates for three-machine robotic flow shops [3]. Crame et al. [4] also presented a survey of cyclic robotic scheduling problem along with their existing solution approaches. Kats and Levner [5] [6] considered  $m$ -machine production line processing identical parts served by a mobile robot to find the minimum cycle time for 2-cyclic schedules. Maimon et al. [7] introduced a neural network method for a material-handling robot task-sequencing problem. Suárez and Rosell [8] dealt with the particular real case of feeding sequence selection in a manufacturing cell consisting of four parallel identical machines. Several feeding strategies and simulation model were built to select the best sequence. Most of the work and theory foundation considered scheduling robots which are usually inflexible, move on prescribed path and repeatedly perform a limited sequence of activities. There is still lack of scheduling mobile robots which are able to move around within a manufacturing cell to process extended logistic tasks. Therefore, in this paper we focus on scheduling that kind of mobile robot for feeding tasks which could be pre-determined based on maximum and minimum levels of parts in feeders.

The remainder of this paper is organized as follows: in the next section, problem statement is described while the mathematical model is formulated in Section 3. A case study is investigated to demonstrate the result of the proposed model in Section 4. Finally, conclusions are drawn in Section 5.

## 2 Problem Description

Fig. 1 shows a typical layout of the manufacturing cell. In particular, the work is developed for a real cell that produced parts for the pump manufacturing industry at a factory in Denmark.



**Fig. 1.** Layout of the manufacturing cell

The cell has a central storage (part “supermarket”), a single mobile robot with eight-hour battery life, and several production lines which consist of multiple machines fed by multiple feeders. The robot carries several small load carriers (SLCs) containing parts from the storage, feeds all parts inside each SLC to each feeder, then returns to the storage to unload all empty SLCs and take filled SLCs. The feeders have to be served a number of times in order to keep producing all of products, so the robot has a set of feeding tasks to carry out during production time of products. In order to accomplish all the movements with a smallest consumed amount of battery energy, the total traveling time of the robot is an important objective to be considered. Hence, it is crucial to determine in which way the robot should feed the feeders of machines in order to minimize its total traveling time within the manufacturing cell while preventing the production lines from stopping working.

### 3 Mathematical Formulation

In this study, a mix-integer linear programming (MILP) model is developed to determine optimal route of the mobile robot visiting a number of locations to process feeding tasks. The model is inspired by well-known traveling salesman problem [9] and the  $(s, Q)$  inventory system [10]. The latter is applied to define time windows for the feeding tasks. In practice, the MILP model can be applied to small-scale problems with a few numbers of feeders, products and short planning horizon. Under these scenarios, the MILP model is reasonably fast to give exact optimal solutions, which can be used as reference points to quantify the scale of benefits achieved by a meta-heuristic method further developed. Assumptions, notations and formulation for the MILP model are extensively described in the following sections.

#### 3.1 Assumptions

- A fully automatic mobile robot with eight-hour battery life is considered in disturbance free environment.
- The robot can carry several SLCs at a time.
- All tasks are periodic.
- All tasks are assigned to the same robot.
- All tasks are independent which means that there are no precedence constraints.
- Working time and traveling time of the robot between any two locations, in which either one of the locations can be a feeder or storage place, are known.
- Consuming rate of parts in a feeder is known.
- All of feeders of machines have to be fed up to maximum level and the robot starts from the storage at the initial stage.

#### 3.2 Notations

$N$  : set of all tasks (0: index of task at the storage)

$n_i$  : total number of times which task  $i$  has to be executed

- $R$  : maximum number of route ( $R = \sum_{i \in N \setminus \{0\}} n_i$ )  
 $e_{ik}$  :  $k$ -th released time of task  $i$   
 $d_{ik}$  :  $k$ -th due time of task  $i$   
 $p_i$  : period time of task  $i$   
 $w_i$  : working time per SLC of task  $i$  ( $w_0$ : working time at the storage)  
 $t_{ij}$  : traveling time of robot from task  $i$  location to task  $j$  location  
 $c_i$  : consuming rate of parts in feeder at task  $i$  location  
 $v_i$  : minimum level of parts in feeder at task  $i$  location  
 $u_i$  : maximum level of parts in feeder at task  $i$  location  
 $Q$  : maximum number of SLCs that can be carried by the robot

Decision variables:

$$x_{ik}^{jlr} = \begin{cases} 1 & \text{if robot travels from } k\text{-th task } i \text{ location to } l\text{-th task } j \text{ location in the route } r \\ 0 & \text{otherwise} \end{cases}$$

- $y_{ik}$  : route number in which  $k$ -th task  $i$  belongs  
 $s_{ik}$  :  $k$ -th starting time of task  $i$

### 3.3 Mixed Integer Linear Programming Model

$$\text{Objective function: } \min \sum_{i \in N} \sum_{k=1}^{n_i} \sum_{j \in N} \sum_{l=1}^{n_j} \sum_{r \in R} t_{ij} x_{ik}^{jlr} \quad (1)$$

Subject to:

$$p_i = (u_i - v_i)c_i \quad \forall i \in N \setminus \{0\} \quad (2)$$

$$e_{i_{k+1}} = e_{ik} + p_i \quad \forall i \in N \setminus \{0\}, k = 1 \div n_i \quad (3)$$

$$d_{ik} = e_{ik} + (v_i - 0)c_i \quad \forall i \in N \setminus \{0\} \quad (4)$$

$$e_{ik} \leq s_{ik} \leq d_{ik} \quad \forall i \in N \setminus \{0\}, k = 1 \div n_i \quad (5)$$

$$\sum_{j \in N \setminus \{0\}} \sum_{l=1}^{n_j} x_{01}^{jll} = 1 \quad (6)$$

$$\sum_{j \in N \setminus \{0\}} \sum_{l=1}^{n_j} \sum_{r \in R} x_{01}^{jlr} \leq 1 \quad (7)$$

$$\sum_{i \in N} \sum_{k=1}^{n_i} x_{ik}^{ikr} = 0 \quad \forall r \in R \quad (8)$$

$$\sum_{r \in R} x_{ik}^{jlr} \leq |Z| - 1 \quad \forall i, j \in N, k = 1 \div n_i, l = 1 \div n_j, i \neq j, Z \subseteq Z_T, Z \neq \Phi \quad (9)$$

$$\sum_{j \in N} \sum_{l=1}^{n_j} \sum_{r \in R} x_{ik}^{jlr} = 1 \quad \forall i \in N \setminus \{0\}, k = 1 \div n_i \quad (10)$$

$$\sum_{i \in N} \sum_{k=1}^{n_i} \sum_{r \in R} x_{ik}^{jlr} = 1 \quad \forall j \in N \setminus \{0\}, l = 1 \div n_j \quad (11)$$

$$\sum_{i \in N} \sum_{k=1}^{n_i} \sum_{j \in N \setminus \{0\}} \sum_{l=1}^{n_j} x_{ik}^{jlr} \leq Q \quad \forall r \in R \quad (12)$$

$$s_{ik} + \left( w_i + t_{ij} \sum_{r \in R} x_{ik}^{jlr} \right) - L \left( 1 - \sum_{r \in R} x_{ik}^{jlr} \right) + (y_{jl} - y_{ik}) \times (t_{i0} + w_0 + t_{0j} - t_{ij}) \leq s_{jl} \quad (13)$$

$$\forall i, j \in N, k = 1 \div n_i, l = 1 \div n_j, \forall r \in R$$

$$y_{jl} = \sum_{i \in N} \sum_{k=1}^{n_i} \sum_{r \in R} r \times x_{ik}^{jlr} \quad \forall j \in N \setminus \{0\}, l = 1 \div n_j \quad (14)$$

$$y_{jl} \geq y_{ik} \sum_{r \in R} x_{ik}^{jlr} \quad \forall i, j \in N, k = 1 \div n_i, l = 1 \div n_j \quad (15)$$

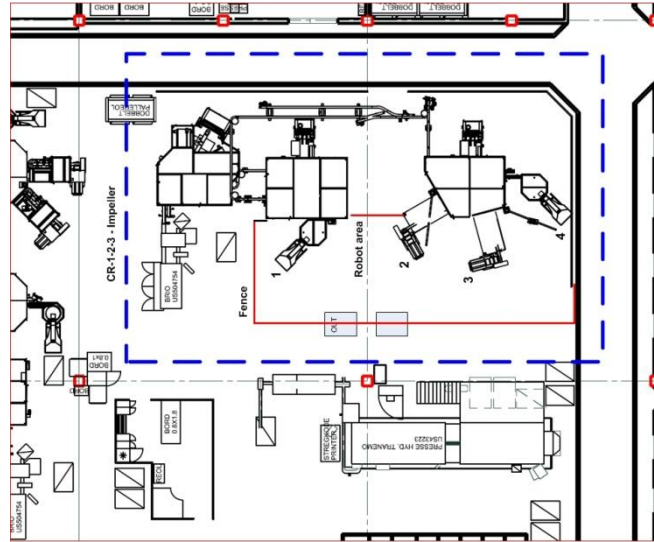
$$x_{ik}^{jlr} \in \{0, 1\} \quad \forall r \in R, \forall i, j \in N, k = 1 \div n_i, l = 1 \div n_j \quad (16)$$

$$y_{ik} : \text{positive integer variable} \quad \forall i \in N, k = 1 \div n \quad (17)$$

The objective is to minimize the total traveling time of robot. Constraints (2), (3), and (4) set period time of each task, released time and due time of each execution of each task, respectively. Constraint (5) ensures that starting time of each execution of each task satisfies its time window. Constraints (6) and (7) indicate that the robot starts from storage at the initial stage. Constraint (8) prevents the robot repeating an execution of one task. Constraint (9) eliminates the sub-tours among executions of tasks, where  $Z$  is a subset of  $Z_T$ , where  $Z_T$  is a set of all executions of tasks at feeders and the storage, and  $\Phi$  denotes an empty set. Constraints (10) and (11) force each execution of each task in one route to be done exactly one. Constraint (12) forbids the robot to feed more SLCs than the maximum number of SLC  $Q$  it allows to carry. Constraint (13) handles the traveling time requirements between any pair of executions of tasks, where  $L$  is a given sufficiently large constant. In case two executions of the same task or different tasks are connected but they are not in the same route, the robot should visit the storage to unload empty SLCs and load filled ones. Constraint (14) assigns each execution of each task to a route and constraint (15) guarantees the ascending sequence of route numbers for executions of tasks. Constraints (16) and (17) imply the types of variables.

## 4 Case Study

To examine performance of the MILP model, a case study is investigated at the CR factory at Grundfos A/S. The chosen area for this case study is the CR 1-2-3 impeller production line which produces impellers for the CR products. The CR line consists of four feeders that have to be served by the mobile robot. These feeders are indexed from 1 to 4 and named Back Plate, Van Feeder 1, Van Feeder 2, and Front Plate respectively. Besides, different feeders are filled by different kinds of parts, namely back plates for feeder 1, vanes for feeder 2 and 3, front plates for feeder 4. On the CR line, a number of vanes are welded together with back and front plates to produce an impeller. Fig. 2 below particularly illustrates the aforementioned production area where the presented model has been implemented in the company.



**Fig. 2.** CR 1-2-3 impeller production line

The maximum number of SLCs carried by the robot is 2. The average number of parts per SLC fed to feeder 1 or 4 is 125 (approximately 2 kg/SLC), while the average number of parts per SLC fed to feeder 2 or 3 is 1100 (approximately 1 kg/SLC). The maximum and minimum levels and consuming rate of parts in each feeder are given in Table 1, while Tables 2 and 3 show working time of robot at each task's location and traveling time of robot from one task's location to another.

**Table 1.** Maximum and minimum levels and consuming rate of parts in feeders

Feeder	1	2	3	4
Maximum level (part)	250	2000	2000	250
Minimum level (part)	125	900	900	125
Consuming rate (sec/part)	4.5	1.5	1.5	4.5

**Table 2.** Working time of robot at feeders and storage

Feeder	0 (storage)	1	2	3	4
Working time (sec)	90	42	42	42	42

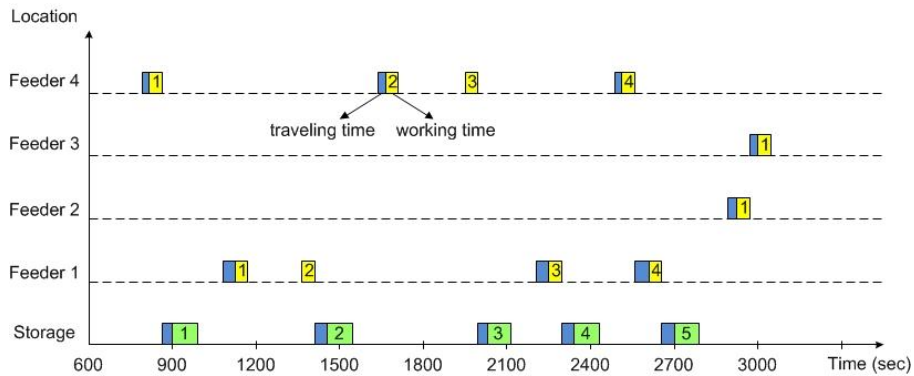
**Table 3.** Traveling time of robot from one location to another

Traveling time (sec)	0	1	2	3	4
0	0	49	44	43	38
1	49	0	58	45	58
2	46	58	0	35	48
3	42	43	35	0	47
4	44	56	47	46	0

The case study was investigated during nearly an hour because of robot’s battery limitation. The MILP model has been coded in the mathematical modeling language ILOG OPL 3.6. The problem of case study has been run on a PC that has Core 2 Duo CPU and 2.2 GHz processor (2 GB RAM). The optimal solution obtained is given as: 0 – 4 – 0 – 1 – 1 – 0 – 4 – 4 – 0 – 1 – 0 – 4 – 1 – 0 – 2 – 3 – 0, with total traveling time being 624 seconds which makes up 20.5% of the total time. With 30530 decision variables, the CPU time for this case using the proposed model is 184s. The detailed solution is shown in Table 4 and Fig. 3 below.

**Table 4.** Detailed optimal solution of the case study

Task	Feeder	Index of execution	Starting time	Route
1	4	1	810.0	1
2	1	1	1125.0	2
3	1	2	1378.5	2
4	4	2	1687.5	3
5	4	3	1935.0	3
6	1	3	2250.0	4
7	4	4	2510.0	5
8	1	4	2608.0	5
9	2	1	2923.0	6
10	3	1	3000.0	6



**Fig. 3.** Gantt chart for optimal solution of the case study

The above optimal solution is an initial schedule for the robot. That schedule serves as an input to a program called Mission Planner and Control (MPC) which is implemented in VB.NET. The MPC program is accessed using XML-based TCP/IP communication to command and get feedbacks from the robot. During the practical feeding operations at CR 1-2-3 impeller production line, the initial schedule was executed in sequence and it prevented all of feeders running out of parts. Hence, the CR line can keep producing impellers without shortage of parts fed from feeders.

## 5 Conclusions

In this paper, a new problem of scheduling a single mobile robot for feeding tasks in a manufacturing cell is studied. To accomplish all tasks within allowable limit of battery capacity, it is important for mission planner to determine optimal feeding sequence to minimize total traveling time of the mobile robot while taking into account specific features of the robot and a number of technological constraints. A new mix-integer linear programming model is developed to find optimal solution for the problem. A particular real case of the impeller production line composing of four feeders is described to show result of the proposed model. The result was quite properly applied during practical feeding operations and it demonstrated that all feeders had no shortage of parts fed to the production line.

For further research, the complexity of the problem categorized as being NP-hard will increase when considering a large number of feeders and/or long planning horizon. Hence, a meta-heuristic method will be taken into account for solving the large-scale mobile robot scheduling problem. Besides, a re-scheduling mechanism based on obtained schedules and feedback from the shop floor will be developed to deal with real-time disturbances.

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